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# PROGRAMME & ABSTRACTS

## Elasticity Day 2022

19 May 2022

University College London



**Lecture Theatre G13**, 1-19 Torrington Pl, London, WC1E 7HB

 **zoom** Livestreaming

**Organised by:**

Dr. Federico Bosi  
*Dept. of Mechanical Engineering*

Dr. Ilia Kamotski  
*Dept. of Mathematics*

 **UCL**

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## Elasticity Day 2022 - Thursday 19 May

All talks will take place in the [Lecture Theatre G13, 1-19 Torrington Pl, London, WC1E 7HB](#)

Lunch and coffee will be provided in the foyer outside G13 and in the lecture room G10

*9:30 - 9:40 Welcome and Introduction*

9:40 - 10:00 [Michel Destrade](#), Artur L. Gower, Guo-Yang Li

**Acoustic evaluation of stresses and strains in soft materials**

10:00 - 10:20 [Yibin Fu](#), Mi Wang, Lishuai Jin

**Axisymmetric necking versus Treloar-Kearsley instability in a hyperelastic sheet under equibiaxial stretching**

10:20 - 10:40 [Tom Shearer](#)

**Bayesian inference on a microstructural, hyperelastic model of tendon deformation**

10:40 - 11:00 [Erik García Neefjes](#)

**Linear thermo-visco-elastic effects in fluid-filled slits and fluid-loaded plates including stress relaxation**

*11:00 - 11:30 Coffee Break*

11:30 - 11:50 [Gregory J. Chaplain](#), Jacopo M. De Ponti, Richard V. Craster, Timothy A. Starkey  
**Twist and Shout: Exciting Spiral Acoustic Waves using Pipes**

11:50 - 12:10 [Alexander B. Movchan](#), Natasha V. Movchan, Ian S. Jones, Graeme W. Milton, Hoai M. Nguyen

**Dynamic Response and Wave Transmission Across Temporal Laminates and Imperfect Chiral Interfaces**

12:10 - 12:30 [Lorenzo Morini](#), Zafer G. Tetik, Zhijiang Chen, Massimiliano Gei

**Universal spectrum representation and band-gap optimisation of quasicrystalline-generated structured rods**

12:30 - 12:50 [Bryn Davies](#)

**Fractal edge modes in quasicrystals: from Fibonacci to Mandelbrot**

*12:50 - 14:00 Lunch Break*

14:00 - 14:20 [John C. Chapman](#)

**A Poisson scaling approach to backward wave propagation in an elastic tube**

14:20 - 14:40 [Mike Smith](#)

**Scattering by a cylindrical Helmholtz resonator**

14:40 - 15:00 [Richard Wiltshaw](#)

**Singular Green's functions for Bloch wave propagation through thin elastic plates loaded with arrays of resonators**

15:00 - 15:20 Mungo G. Aitken  
**Diffraction of SH Waves by a Thick Semi-Infinite Crack**

15:20 - 15:40 Kevin Jose, Neil Ferguson, Atul Bhaskar  
**Branched flows of flexural waves in elastic plates and cylindrical shells**

15:40 - 16:10 *Coffee Break*

16:10 - 16:30 Valery Smyshlyaev  
**High-contrast periodic elastic beam lattice materials: micro-resonances, bandgaps, error estimates and generalisations**

16:30 - 16:50 Giulia Aguzzi, Andrea Colombi, Eleni N. Chatzi  
**Octet-like reticulated structures for attenuation and guiding of elastic waves**

16:50 - 17:10 Isaac V. Chenchiah, Matthew P. O'Donnell  
**Morphing Hexagonal Frameworks and Meta-Materials**

17:10 - 17:30 Melle Gruppelaar, Eral Bele and PJ Tan  
**Effects of strut-buckling on the fracture toughness of finite-sized triangular lattices**

17:30 - 17:40 *Closing Remarks*

# Acoustic evaluation of stresses and strains in soft materials

Michel Destrade<sup>1,2</sup>, Artur L. Gower<sup>3</sup>, Guo-Yang Li<sup>4</sup>

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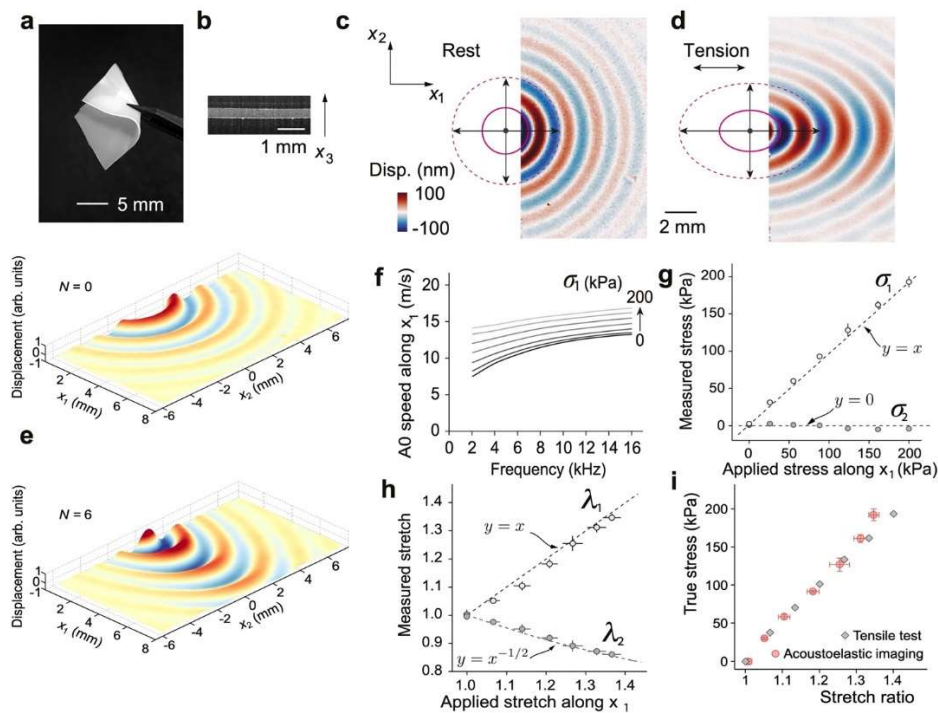
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With this talk we will see that the states of stress and strain in a loaded material can be accessed directly from wave speed measurements, without having to determine, or even know, its material properties beyond its mass density and geometry. These techniques presented are expected to have important applications in the health monitoring of loaded structures. Examples include stressed rail steel, the human skin in situ, and thin membranes such as a stretched rubber sheet, a piece of cling film (~10  $\mu\text{m}$  thick) and the animal skin of a bodhrán, a traditional Irish drum.



**Figure: Imaging and analysing elastic waves in a stretched rubber membrane.** **a**, Photograph of our rubber sheet. **b**, Optical Coherence Tomography (OCT) cross-sectional image. **c,d**, Wave profiles measured by OCT when the film is **(c)** stress-free and **(d)** subject to uniaxial stress  $\sigma_1 \approx 200$  kPa ( $N = 6$  weights of 20 g each added). The wavefronts become elliptical with the stretch. **e**, Corresponding 3D wavefronts, obtained by measuring the vertical displacement from the phase change in the interference signal of the OCT. **f**, Dispersion relations of the AO mode obtained at different levels of stress. **g**, Comparison between measured and applied stress values. For reference, the dashed lines show where the measured stresses are equal to the applied stresses. **h**, Comparison between the measured and applied stretch values. Dashed lines, expected 1 and  $-1/2$  power laws of the applied stretch for reference. **i**, Stress-strain curve from the measured data compared to that measured by a standard tensile test.

# Axisymmetric necking versus Treloar-Kearsley instability in a hyperelastic sheet under equibiaxial stretching

Yibin Fu<sup>1</sup>, Mi Wang<sup>2</sup>, Lishuai Jin<sup>3</sup>

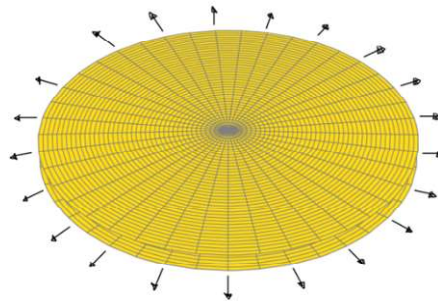
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We consider bifurcations from the homogeneous solution of a circular or square hyperelastic sheet that is subjected to equibiaxial stretching under either force- or displacement-controlled edge conditions. We derive the condition for axisymmetric necking and show, for the class of strain-energy functions considered, that the critical stretch for necking is greater than the critical stretch for the Treloar-Kearsley (TK) instability and less than the critical stretch for the limiting-point instability. An amplitude equation for the bifurcated necking solution is derived through a weakly nonlinear analysis and is used to show that necking initiation is generally sub-critical. Abaqus simulations are conducted to verify the bifurcation conditions and the expectation that the TK instability should occur first under force control, but when the edge displacement is controlled the TK instability is suppressed, and it is the necking instability that will be observed.



## References:

[1] R.W. Ogden, "Local and global bifurcation phenomena in plane-strain finite elasticity", *Int. J. Solids Struct.* 21 (1985), 121–132.

[2] E.A. Kearsley, "Asymmetric stretching of a symmetrically loaded elastic sheet", *Int. J. Solids Struct.* 22 (1986), 111–119.

[3] M. Wang, L.S. Jin and Y.B. Fu, "Axisymmetric necking versus Treloar-Kearsley instability in a hyperelastic sheet under equibiaxial stretching", *Math. Mech. Solids*, 2022.

<https://doi.org/10.1177/10812865211072897>

# **Bayesian inference on a microstructural, hyperelastic model of tendon deformation**

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Microstructural models of soft tissue deformation are important in applications including artificial tissue design and surgical planning. The basis of these models, and their advantage over their phenomenological counterparts, is that they incorporate parameters that are directly linked to the tissue's microscale structure and constitutive behaviour and can therefore be used to predict the effects of structural changes to the tissue. Although studies have attempted to determine such parameters using diverse, state-of-the-art, experimental techniques, values ranging over several orders of magnitude have been reported, leading to uncertainty in the true parameter values and creating a need for models that can handle such uncertainty. In this talk, we derive a microstructural, hyperelastic model for transversely isotropic soft tissues and use it to model the mechanical behaviour of tendons. To account for parameter uncertainty, we employ a Bayesian approach and apply an adaptive Markov chain Monte Carlo algorithm to determine posterior probability distributions for the model parameters. The obtained posterior distributions are consistent with parameter measurements previously reported and enable us to quantify the uncertainty in their values for each tendon sample that was modelled. This approach could serve as a prototype for quantifying parameter uncertainty in other soft tissues.

# Linear thermo-visco-elastic effects in fluid-filled slits and fluid-loaded plates including stress relaxation

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A recently developed framework for linear thermo-visco-elastic (TVE) continua allows us to conveniently study waves in a 2D configuration consisting of a layer of TVE material in perfect contact with a different TVE material separated by two (infinitely extending) parallel interfaces. Despite the validity of the media described by the theory we want to mainly investigate fluid—solid interfaces, so we show that the corresponding dispersion equations for symmetric/antisymmetric modes to this system allows us to consider thermo-viscous effects in fluid-filled ducts as well as fluid-loaded plates. In the short wavelength limit (relative to the slit/plate width) the respective modes are governed by the TVE Scholte-Stoneley equation which is discussed first. With this as a starting point, we show how for longer wavelengths the Scholte-type modes become coupled in the slit/plate region and discuss their dispersive behaviour. Most of the numerical results are applied to water and both ‘hard’ and ‘soft’ solids are compared, with emphasis put on the importance of viscoelastic effects, particularly when stress relaxation is considered.



## **Twist and Shout: Exciting Spiral Acoustic Waves using Pipes**

Gregory J. Chaplain<sup>1</sup>, Jacopo M. De Ponti<sup>2</sup>, Richard V. Craster<sup>3</sup>, Timothy A. Starkey<sup>1</sup>

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We present the elastic spiral phase pipe, a structure capable of exciting guided flexural waves that carry orbital angular momentum (OAM). The transfer of the elastic OAM associated with such waves can be achieved through the coupling of ultrasonic waves in the pipe with the pressure field in a fluid, in which the pipe is partially submerged; the circumferential distribution of the stress in the pipe act as a continuous phased pressure source in the fluid, resulting in the generation of Bessel-like acoustic beams. We observe this transfer via simulation that is corroborated experimentally.



# Dynamic Response and Wave Transmission Across Temporal Laminates and Imperfect Chiral Interfaces

Alexander B. Movchan<sup>1</sup>, Natasha V. Movchan<sup>1</sup>, Ian S. Jones<sup>1,2</sup>,  
Graeme W. Milton<sup>3</sup>, Hoai Minh Nguyen<sup>4</sup>

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The lecture includes analysis of wave patterns in a structure which possesses periodicity in the spatial and temporal dimensions. A special attention is given to the topic of imperfect chiral interfaces. Although causality is fundamental for physical processes, natural wave phenomena can be observed when a wave is split at a temporal interface. In particular, when the coefficients of the governing equations are time-dependent, the temporal interface becomes important. For a class of transient problems, the frontal edge waves are studied in detail, and regimes are analysed where the growth of the solution in time is observed. Imperfect interfaces, across which the displacements are discontinuous, are also considered in the vector case of chiral elastic systems.

The analytical asymptotic approximations of solutions are also illustrated with numerical examples.

# Universal spectrum representation and band-gap optimisation of quasicrystalline-generated structured rods

Lorenzo Morini<sup>1</sup>, Zafer G. Tetik<sup>1</sup>, Zhijiang Chen<sup>1</sup>, Massimiliano Gei<sup>2</sup>

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We investigate the dynamical properties of a particular class of periodic two-component phononic rods, whose elementary cells are generated adopting the Fibonacci substitution rules, by means of the recently introduced method of the toroidal manifold [1]. The method allows all band gaps and pass bands featuring the frequency spectrum to be represented in a compact form with a frequency-dependent flow line on the surface describing their ordered sequence. The flow lines on the torus can be either closed or open: in the former case, (i) the frequency spectrum is periodic and the elementary cell corresponds to a canonical configuration [2], (ii) the band gap density depends on the lengths of the two phases; in the latter, the flow lines cover ergodically the torus and the band gap density is independent of those lengths. It is then shown how the proposed compact description of the spectrum can be exploited (i) to find the widest band gap for a given configuration and (ii) to optimize the layout of the elementary cell in order to maximize the low-frequency band gap [3]. The scaling property of the frequency spectrum, that is a distinctive feature of quasicrystalline-generated phononic media [4], is also confirmed by inspecting band-gap/pass-band regions on the torus for the elementary cells of different Fibonacci orders.

## References:

- [1] Shmuel G. and Ram B. Universality of the frequency spectrum of laminates, *J. Mech. Phys. Solids*, 92 (2016) 127-136.
- [2] Gei M., Chen Z., Bosi F. and Morini L. Phononic canonical quasicrystalline waveguides, *Appl. Phys. Lett.*, 116 (2020) 241903.
- [3] Morini L., Tetik Z.G., Shmuel G. and Gei M. On the universality of the frequency spectrum and band-gap optimisation of quasicrystalline-generated structured rods, *Phil. Trans. R. Soc. A*, 378 (2020) 20190240.
- [4] Morini L. and Gei M. Waves in one-dimensional quasicrystalline structures: dynamical trace mapping, scaling and self-similarity of the spectrum, *J. Mech. Phys. Solids*, 119 (2018) 83-103.

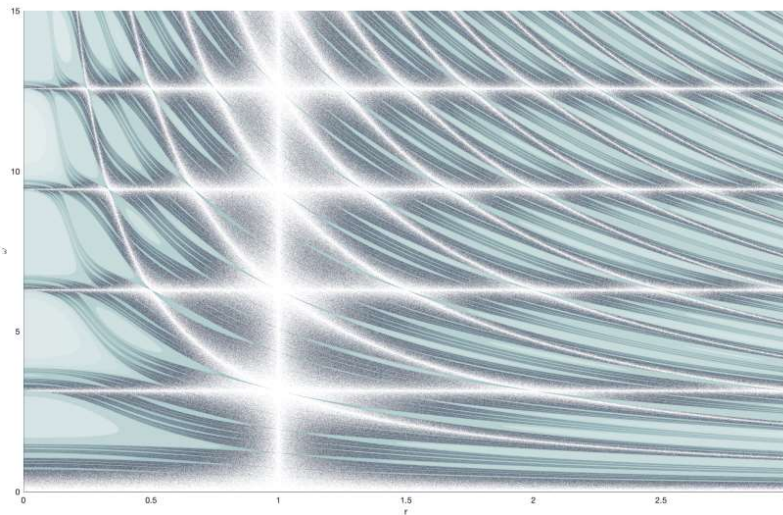
# Fractal edge modes in quasicrystals: from Fibonacci to Mandelbrot

Bryn Davies

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In this talk, we show how introducing an axis of reflectional symmetry in a quasicrystal leads to the creation of fractals of localised edge modes. We model axial and flexural waves in quasiperiodic infinite beams based on the Fibonacci sequence, which have been studied previously and shown to have exotic, Cantor-like spectra with very wide band gaps. Our approach provides a way to create localised edge modes at frequencies within these band gaps, giving robust and broadband wave localisation. More broadly, we have developed a general method for modelling reflection-induced modes in media defined by recursion relations. This has implications for periodic and random media, as well as quasicrystals, and reveals a partial link with the Mandelbrot set.



**Figure:** The spectrum of a medium based on the Fibonacci sequence that has been reflected to create an interface. Coloured regions are spectral band gaps, with the colour chosen according to how quickly localised modes decay at that frequency.  $r$  is a material parameter and  $r = 1$  is the trivial case, where the medium is homogeneous.

# **A Poisson scaling approach to backward wave propagation in an elastic tube**

John C. Chapman

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This talk analyses the dispersion relation for wave propagation at negative group velocity in an elastic tube, which may be thin- or thick-walled. It involves the mathematical idea of dominant balance, used here to determine a distinguished scaling involving Poisson's ratio and the thickness of the wall. By comparison with numerical results obtained to machine precision, the asymptotic formulae obtained from this scaling are shown to have extraordinary accuracy, and suggest the possibility a new type of shell theory, which may be called Poisson-scaled shell theory. The new approach is especially adapted to thick as well as thin shells.

## *References:*

[1] C. J. Chapman & S. V. Sorokin (2022). A Poisson scaling approach to backward wave propagation in a tube. *Phil. Trans. Roy. Soc. A.* doi:10.1098/rsta.2022.0386

# Scattering by a cylindrical Helmholtz resonator

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Helmholtz resonators are an important design element in the development of metamaterials and advanced composite materials, specifically, playing a key role in the manufacture of photonic, phononic, and other crystals. They are also used in a range of wave scattering models, where careful arrangements of resonators can be employed to enhance object cloaking, to trap wave energy, and even guide incident wave energy away from critical regions. To date, a broad selection of two- and three-dimensional resonator designs have been studied in an extensive literature dating back to the 19th century with the works of Lord Rayleigh and Helmholtz. However, the majority of articles in recent decades have obtained results using purely numerical methods, with a comparatively limited range of analytical treatments available for crucial insight. In this talk, I will discuss an elegant closed-form treatment for describing plane wave scattering by a single two-dimensional Helmholtz resonator. Analytical representations for the scattering cross-section and extinction are obtained using a combination of multipole methods and the method of matched asymptotic expansions. I will present analytic expressions for the scattering amplitudes which reveals unambiguously how the geometry of the resonator can be tuned to achieve low-frequency resonances, which has important implications for scattering models and metamaterials design.

# **Singular Green's functions for Bloch wave propagation through thin elastic plates loaded with arrays of resonators**

Richard Wiltshaw

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We consider the analytical solution of wave scattering by periodic arrays of beams attached to the surface of a thin elastic plate. The Green's functions are expressed utilizing either Fourier series or transforms. The solutions are readily exploited to create rapid and accurate numerical codes, computing the dispersion relations for Floquet-Bloch wave propagation through an infinitely large periodic array of scatterers; additionally, scattering problems concerning finite arrays excited by some incident source can be computationally solved. The solutions will be demonstrated with some interesting topological examples.

# Diffraction of SH Waves by a Thick Semi-Infinite Crack

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This talk concerns the diffraction of SH waves by a thick semi-infinite crack, which may be reformulated as a problem of acoustic diffraction by a rigid bar of finite thickness. This canonical problem was first studied by Jones (1953), and later by Crighton and Leppington (1973), and is a natural generalization of the classical Sommerfeld problem, which is diffraction by a semi-infinite screen of zero-thickness. The results obtained by Crighton and Leppington, valid only at low frequency, or equivalently for small plate thickness, verified Jones' results in this limit.

We revisit Jones' problem and offer a new method of solution, wherein we formulate it as a matrix Wiener-Hopf equation, which can be tackled via the pole removal method. Convergence of the resulting linear algebraic system of equations in the Fourier domain can be significantly enhanced by 'building-in' the behaviour of the physical field in the corner regions. The final result is remarkably elegant in its simplicity and moreover is valid over all frequencies. As time allows, we will present some polar plots of the diffracted field in the near and far fields.



# **Branched flows of flexural waves in elastic plates and cylindrical shells**

Kevin Jose, Neil Ferguson, Atul Bhaskar

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Here we report the branching behaviour of directed elastic waves as they are launched at one end of a plate or a cylindrical shell, when the medium possesses modest variations in its bending stiffness. The correlation length of the inhomogeneity of the medium is much greater than the wavelength. Branched flows refer to the surprising emergence of branching and extreme amplitude values in waves carried by media with weakly random parameters. Associated with this, is also an elegant scaling of the expected location of the first extreme amplitude events with correlation length and severity of randomness. Branched flows have been previously shown to exist in various physical media and length scales ranging from microwaves to tsunamis. We demonstrate this analytically – starting from plate and shell dynamics equations that ignore higher order effects such as thickness shear and rotary inertia – by using paraxial ray approximations and stochastic calculus that leads to the relevant Fokker-Planck equations for the temporal evolution of the probability density. Numerical integration of the ray equations and FEA simulation of the full wave dynamics show a scaling that is consistent, the exponent of which appears to have a universality.

# High-contrast periodic elastic beam lattice materials: micro-resonances, bandgaps, error estimates and generalisations

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In [1] we studied wave propagation in a two-dimensional periodic lattice network of Timoschenko-type elastic beams. For configurations involving certain highly contrasting components a high-contrast modification of the homogenization theory is capable of accounting for bandgaps, explicitly relating those to low resonant frequencies of the “soft” components. This fits as a particular case into our recent work [2] on a general scheme for two-scale type asymptotic approximations with controllable tight errors of “operator-type”. As a result, not only an asymptotically explicit description of the band gaps is given but also error estimates between the original and effective spectra can be established. The approach allows to treat a much wider class of “degenerating” problems, from elasticity and beyond.

## *References:*

- [1] Kamotski, I.V., Smyshlyaev, V.P., Bandgaps in two-dimensional high-contrast periodic elastic beam lattice materials, *J. Mech. Phys. Solids*, 123, 292-304 (2019).
- [2] Cooper S., Kamotski I.V., Smyshlyaev V.P., Uniform asymptotics for a family of degenerating variational problems with applications to error estimates in homogenisation theory". Preprint.

# Octet-like reticulated structures for attenuation and guiding of elastic waves

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In a plethora of frame-based lattices, we here focus on the well-known octet topology [1] for the purpose of attenuation and guiding of elastic waves. Owing to its high strength-to-density ratio, this comprises an extensively exploited class of truss-like structures in engineering applications; however, less is known about its potential in controlling elastic waves. In this study, we delve into the inherent dynamics of the octet lattice by analyzing its dispersion relations and demonstrate the initiation of a wide bandgap resulting from the bending resonance of its struts. With its underlying physics revealed, we readily identify the parameters governing the frequency content of its stop band and perform a sensitivity analysis. Eventually, we combine this topology with the grading concept [2] to numerically design a reticulated plate equipped with two devices for elastic wave attenuation and guiding, a metabarrier and a metalens. Our results verify that these structures effectively manipulate the trajectory of propagating waves in the targeted frequency range [3]. In addition, they prompt attention for further investigations on experimental validation and deeper insights as to nonlinear effects of the struts.

## *References:*

- [1] Deshpande, V. S., Fleck, N. A., & Ashby, M. F. (2001). Effective properties of the octet-truss lattice material. *Journal of the Mechanics and Physics of Solids*, 49(8), 1747-1769.
- [2] Colombi, A., Craster, R. V., Colquitt, D., Achaoui, Y., Guenneau, S., Roux, P., & Rupin, M. (2017). Elastic wave control beyond band-gaps: shaping the flow of waves in plates and half-spaces with subwavelength resonant rods. *Frontiers in Mechanical Engineering*, 3, 10.
- [3] Aguzzi, G., Kanellopoulos, C., Wiltshaw, R., Craster, R. V., Chatzi, E. N., & Colombi, A. (2022). Octet lattice-based plate for elastic wave control. *Scientific Reports*, 12(1), 1-14.

# Morphing Hexagonal Frameworks and Meta-Materials

Isaac V. Chenchiah<sup>1</sup>, Matthew P. O'Donnell<sup>2</sup>

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We consider planar hexagonal frameworks comprised of bi-stable one-dimensional elements -- i.e. geometrically, we consider a hexagonal lattice, each of whose edges can take one of two possible lengths. We analyse the zero-energy (stress-free) states of such a lattice and explore low elastic-energy transition pathways between such states. Our results apply both to morphing structures and to meta-materials.

# Effects of strut-buckling on the fracture toughness of finite-sized triangular lattices

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Microlattices have attracted significant interest as lightweight, resource-efficient structural materials. One of their notable advantages is the ability to attain high weight-specific mechanical properties by manipulating how the material is distributed in space. Their fracture toughness ( $K_{IC}$ ) has been shown to scale with relative density ( $\bar{\rho}$ ) according to a power-law relationship of the form  $K_{IC} = D\bar{\rho}^d\sigma_0\sqrt{l_0}$  [1], where  $\sigma_0$  is the cell wall fracture stress and  $l_0$  is the characteristic cell size. However, this relationship can break down when nonlinear deformation mechanisms, such as buckling, are present. The effect of strut buckling is analysed in triangular lattices with a standard fracture toughness specimen geometry (SEN-T), and a mismatch between the scaling law prediction and the measured fracture toughness is revealed. It is demonstrated that stress bifurcation in struts near the crack-tip occurs below a critical transition relative density. The test-specimens show fracture initiation in the post-buckling regime with fracture toughness up to 60% below the standard scaling law. In deeply notched finite-sized specimens ( $a/W > 0.5$ ), buckling and fracture are observed at the remote edge from the crack. Experimental results confirm the occurrence of strut buckling before fracture in SEN-T specimens and validate the numerically obtained fracture toughness scaling.

## References:

- [1] N.E.R. Romijn and N.A. Fleck, The fracture toughness of planar lattices: Imperfection sensitivity, *Journal of the Mechanics of Physics and Solids* 55: 2538-2564, 2007.

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